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Location-selection of Wireless Network Based on Restricted Steiner Tree Algorithm

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Abstract

The minimum Steiner tree problem has wide application background, such as transportation system, communication network, pipeline design and VISL, etc. It is unfortunately that the computational complexity of the problem is NP-hard. People are common to find some special problems to consider. In this paper, we introduce the definition of restricted Steiner tree problem, i.e., a restricted Steiner tree problem, which the fixed vertices are in the same side of one line L and we find a vertex on L such the distance of the tree is minimal. By the definition and the complexity of the Steiner tree problem, we know that the complexity of this problem is also Np-complete. Therefore, we consider there are two or three fixed vertices to find the restricted Steiner tree problem. We use the algorithm of restricted Steiner tree problem to solve the location-selection of wireless network.

Keywords: restricted Steiner tree, computational complexity, location-selection, wireless network.

1. Introduction

On building communicating network problem, people should be considered in the construction of some cities (site) how lay cable, to make these cities become a connected networks and at the same time to consider how to minimize the cost of the total attachment in traffic. It has the similar problem in transmission system. Since the 1950s, information science has the vigorous development, such that promoting the shortest connection of this kind of problem, formed the research field of a very active combination optimization's topic. There are too much models on this area, roughly divided into two

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categories: the attachment problem in the plane and the network connection problems. For the former, it is considered the connection is the straight line of the plane and the definition of distance is Euclidean distance; For the latter the connection is considered the shortest path in the network, where the distance of two vertices is equals the distance of the shortest path between the vertices.

On location-selection of wireless network , the consumer always consider whether the network is smooth or not. In the common case, the signal of the network bases on the distance between the signal launch and consumer. If the distance is closer, then the signal is stronger. We want to build more launchers but the cost is too expensive. Then we want to find the suitable place to set the launcher such that the sum distance between the launcher and consumers is minimal. The problem is equitable to the famous Steiner tree problem. The computer complexity is NP-hard. In this paper, we consider some special cases of the Steiner tree problem. If we want to build a launcher, then we have to consider the situation of the launcher, such as the geographical environment, the artificial construction etc. Without loss of generality, we demand the launcher in the fixed line L. We assume the launcher must be placed on the fixed line L and the consumers are on the same side of the line L, i.e., the restricted Steiner tree problem [6].

In this paper we consider the connection line problems in the plane. In the plane of the shortest connection problem is divided into two types: the first kind only consider the connection among some fixed vertices and all attachment is constructed the spanning tree of these vertices. Therefore, this problem is called the spanning tree problem. The second is allowed to add some attachment vertices such that the distance of all attachment edges is constituted the shorted network; this is called the shortest Steiner smallest tree problem.

The minimal spanning tree problem is originated from the comprehensive work of Kruskal and Prim. In the plane of the minimum spanning tree problem can be stated as follows: Given a set N which has some fixed vertices and |N|=n in a plane, where the distance of each line (or edge) e=(u,v) is defined the Euclidean distance between the two vertices u and v. Now for a spanning trees of G, namely a connection graph with the vertex set is N to make the sum of edges minimize. Kruskal and Prim has established by the famous Greedy algorithm, even applied to matroid optimization problem. By using the algorithm, the spanning tree problem can be briefly described as follows:

(1) for the graph G, we can sort the distance of the edges;

(2) we choose a smallest edge to construct an edge set;

(3) choose one the one the shortest edge e which is not selected, if e and selected edges does not constitute a circle, then we choose the edge to the edge set, otherwise, we abandon the edge;

(4) the algorithm does not end until we choose n - 1 edges to the edge set, such that we have a smallest spanning tree T of the graph G.

The algorithm time bound of the algorithm is $O(n^2 \log n)$, now there exists more effective polynomial algorithm. Minimum Steiner tree problem is originated from three-point sitting Fermat problem, where there are three villages whose position fitly constitute a triangle, and assumed the ground can be used anywhere for all road’s construction. The problem is how to design a graph G such that the villages are connected and the distance of roads reached the shortest.

The answer to this question is very simple. We can choose one point inside the triangle such that the point attachment angle to the three points are 120^0 . This is an optimal plan (if the maximum angle among the triangle is at least 120^0 , then the optimal plan is the two shorter edges.)

In paper [6] and [7], we have give two algorithms to solve the two cases of the restricted Steiner tree problem. Nextly we use the algorithm to solve the location-selection of wireless network problems.

2. Case for n=2

In the following we consider there two consumers, A and B, are in the same side of the line L. Without loss of generality, we assume the distance of A and L is shorter than the distance of B and L, see the figure 1.

We assume that there is a vertex set N and line L in the Euclidean plane, where the vertices of N are sited in the same side of L. The restricted Steiner tree is denoted by $CS^*(N)$ and the unrestricted Steiner tree is denoted by $S^*(N)$. At the same time the singles are denoted the corresponding edges of the tree. We use the signal AB to denote the edge between vertices of A and B, d(AB) to denote the Euclidean distance of A and B (or by |AB|), l(T) to denote the sum distance of the edges of T.

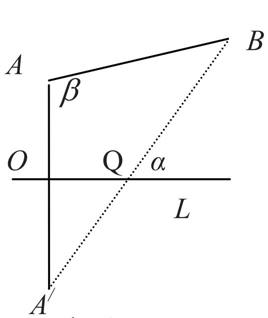

Fig. 1

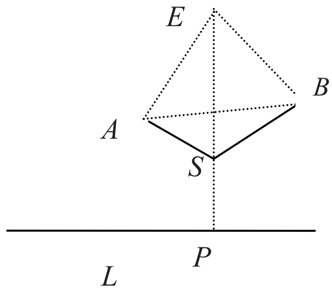

Fig. 2

Let P be the Steiner vertex, and A' is the symmetric vertex of A by the line L. Let α be the cross angle between BA' and L and $\beta = \angle A'AB$. We divide some two cases to consider the choice of P.

Case 1 $\beta < 120^0$.

Case 1.1 $\alpha > 30^0$.

We use the following algorithm:

Step 1 structure the regular triangle ΔABE

Step 2 make a vertical line to L, the cross vertex is denoted by P.

Step 3 Let S be the intersection of EP and inscribed circle of ABE.

The point P is the suitable launcher such the distance between P and two consumers A and B.

Case 1.2 $\alpha \le 30^0$.

Since $\beta < 120^0$. we see that there are two edges incident with P. The suitable launcher P is the cross vertex between A'B and L.

Case 2 $\beta \ge 120^0$.

Under this case, we know that $\alpha > 30^0$. Let Q be the cross vertex between BA' and L. Therefore, O is the location-selection of suitable launcher .

3. Case for n=3

In the following we consider there three consumers, denoted by A, B and C, are in the same side of the line L, see the figure 3. Assumed that the coordinates are (x_1, y_1), (x_2, y_2) and (x_3, y_3), respectively and $y_3 = \min\{y_1, y_2, y_3\} > 0$. In the following we divide two cases to discuss determination of CS*(N): one case is $x_3 = \max\{x_1, x_2, x_3\}$, the other case is $\max\{x_1, x_2, x_3\} > x_3 > \min\{x_1, x_2, x_3\}$.

Let $s^*(N)$ be the optimal Steiner tree determined by A, B and C. Let AD and CE be two edges which are vertical with the line L, where D and E are the cross vertices. Write $s_A(N) = s^*(N) \cup AD$ and $s_C(N)=s^*(N)\cup CE$.

We use a Normalize Rule for $s_A(N), s_C(N)$.

For $s_C(N)$, we do the Normalize Rule.

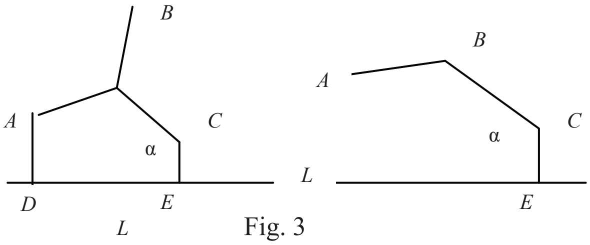

Fig. 3

Let α be the angle between two edges incident with C. Case 1.1 $\alpha \geq 120^0$, $s_C(N)$ is according to the demand of normalization rule, see the figure 3.

Case 1.2 $\alpha < 120^0$.

Let C' be the symmetric vertex of C by the L , G be the cross vertex between $s^*(ABC)C'$ and L, and β be the acute angle according to the cross vertex G. We divide two cases to consider:

Case 1.2.1 $\beta > 30^0$.

Inside the area which is spanning by the angle α, we choose one vertex S such that:

(1) SF is an edge which is vertical with L and the cross vertex is F;

(2) S is the Steiner vertex of $s^*(ABCF)$ and S in incident with F and C in the optimal Steiner tree.

This is the normalization rule of $s_C(N)$, see the figure 2.

Case 1.2.2 $\beta \leq 30^0$.

In this case the normalization rule of $s_C(N)$ is $s^*(ABG) \cup GC$, see the figure 3.

Let $s^*_C(N)$ be the normalization rule of $s_C(N)$. For the normalization rule we have some conclusion:

Proposition 1 For the case 1, where N={A,B,C}, we have $\min\{S^*_A(N), S^*_C(N)\}=CS^*(N)$. Especially, if $s^*_A(N)$ does not exist, then $s^*_C(N) = CS^*(N)$.

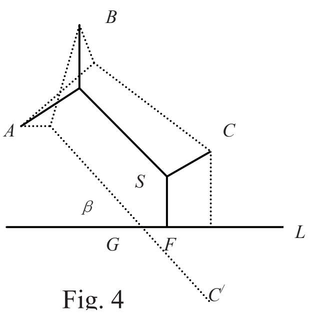

Fig. 4

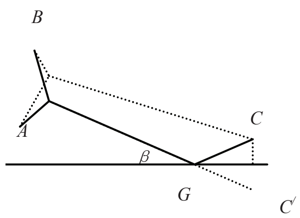

Fig. 5

CASE 2 $\max\{x_1, x_2, x_3\} > x_3 > \min\{x_1, x_2, x_3\}$.

Without loss of generality, we assume that $x_1 = \min\{x_1, x_2, x_3\}$ and $x_2 = \max\{x_1, x_2, x_3\}$.

We introduce one edge CD which is vertical with L and the cross vertex is D.

We have the following normalization rule to attain $s^*_1(N)$:

(1) if $\angle ACD \geq 120^0$, then $s^*_A(N)$ does not exist;

(2) if $\angle ACD < 120^0$, then introduce two terms A' and angle β, where A' is the symmetric vertex of A by the L and β is angle constructed by Ac' and L.

Case 2.1 if $\beta \geq 30^0$, $s^*_1(N) = CS^*(AB) \cup BC$, see the figure 6.

Case 2.2 if $\beta < 30^0$, the optimal Steiner tree $s^*(A'BC)$ must be intersected with L, denoted the cross vertex by G. Let $s^*_1(N) = s^*(BCG) \cup AG$, see the figure 7.

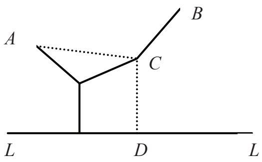

Fig. 6

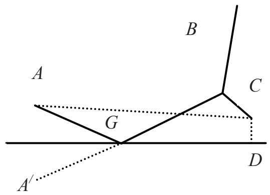

Fig. 7

Similar to the case of P is adjacent to A, if P is adjacent o B, we can definite the $s^*_2(N)$. Let $s_3(N)$ be the set of trees which are constructed by $s^*(N) \cup CD$.

It is easy to check the following facts:

if $\angle ACD \geq 120^0$, then $s^*_1(N)$ does not exist;

if $\angle ACB > 120^0$, then $s^*_3(N)$ does not exist.

Proposition 2 In the case 2, $CS^*(N) = \min\{S^*_1(N), S^*_2(N), S^*_3(N)\}$.

4. Some further problems

We consider the restricted Steiner tree problem for $n = 2$ and $n = 3$ to solve the location-selection of wireless network. It is narural to consider the problem for $n = 4$. Maybe in this case the problem become more complicated. Since for the Steiner tree , $n = 4$ is very different to $n = 3$. We think it will be difficulty to solve. We can also consider the case of $n = 5$ since Yue MinYi and Cheng Congdian give a note on $n = 5$ for steiner tree [5]. It provides a new thought to solve the restricted Steiner tree problem.

Acknowledgements.

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